

Dual chiral density wave in quark matter

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Abstract

Possible manifestation of a *dual chiral density wave* in quark matter is discussed in relation to the chiral symmetry of QCD, which is described by a dual standing wave of the scalar and pseudo-scalar densities. It is demonstrated that quark matter is unstable for forming the dual chiral density wave at moderate densities in the Nambu-Jona-Lasinio model; accordingly the critical density for restoration of chiral symmetry becomes higher than ever considered. A unique magnetic property is also pointed out in the new phase.

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Recently condensed matter physics of QCD has been an exciting area in nuclear physics. Superconductor model of the vacuum is a classic one, firstly considered by Nambu and Jona-Lasinio to understand the nucleon mass [1]. Nowadays it is believed that quark-anti-quark pairs condense in the vacuum, leading to the spontaneous symmetry breaking (SSB) of chiral symmetry in QCD. In the past decade the color superconductivity (quark-quark pair condensation) in quark matter has been extensively studied by many authors [2]. There are also some works to inquire the coexistence of these two phases on the phase diagram of QCD [3].

On the other hand, the possibility of ferromagnetism (FM) in quark matter has been also discussed [4, 5], stimulated by the discovery of magnetars [6]. It has been suggested [4] that ferromagnetic phase exists at rather low densities in analogy with an electron gas discussed by Bloch [7] and demonstrated that relativistic effects play an important role in quark matter. We have also studied a coexistent mechanism of FM and CSC by assuming the spin-parallel pairing within the mean-filed approximation [8].

It should be noted that CSC arises from particle-particle (p - p) correlation, while particle-hole (p - h) correlations should be responsible for SSB of chiral symmetry and FM. Besides, different types of the p - h condensations has been proposed [9, 10] in relation to chiral symmetry, where the scalar or tensor density forms a standing wave with a finite momentum, called the chiral density wave. The instability for the density wave in quark matter was first discussed by Deryagin *et al.* [11] at asymptotically high densities where the interaction is very weak, and they concluded that the density-wave instability prevails over the BCS one in the large N_c (the number of colors) limit due to the dynamical suppression of colored pairing interactions.

We consider here another type of the density wave due to chiral symmetry in QCD. In this letter we are concentrated on the flavor- $SU(2)$ case, and study the flavor-singlet scalar density, $\langle \bar{q}q \rangle$, and the flavor-triplet pseudo-scalar density, $\langle \bar{q}i\gamma_5\tau_i q \rangle$, which transform as the $[1/2, 1/2]$ representation in $SU(2)_L \times SU(2)_R$ chiral symmetry. In the SSB phase, they satisfy the constraint, $\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5\tau_i q \rangle^2 = A^2$ in the chiral limit within the mean-field approximation, and the ground states are degenerate on the hypersphere in the chiral space, prescribed by the scalar and pseudo-scalar mean-fields, if both densities are spatial constants.

We introduce a *dual chiral-density wave* (DCDW) state, where both the scalar and

pseudo-scalar densities always reside on the hypersphere with a constant modulus A , while each density is *spatially non-uniform*; restricting ourselves to the electric charge eigenstate, we consider the following configuration in quark matter,

$$\begin{aligned}\langle\bar{q}q\rangle &= A \cos\theta(\mathbf{r}) \\ \langle\bar{q}i\gamma_5\tau_3 q\rangle &= A \sin\theta(\mathbf{r}).\end{aligned}\tag{1}$$

The chiral angle θ represents the degree of freedom of the Nambu-Goldstone mode or the directional mode on the hypersphere. Taking the simplest but nontrivial form for the chiral angle θ such that $\theta(\mathbf{r}) = \mathbf{q} \cdot \mathbf{r}$, we call this configuration DCDW.

It should be noted that we can construct the DCDW state by acting a space-dependent chiral transformation such as $q \longrightarrow \exp(i\gamma_5\theta(\mathbf{r})/2)q$, on the usual SSB phase where only the scalar density condenses. This is nothing but a kind of Weinberg transformation [12].

When the chiral angle has some spatial dependence, there should appear two extra terms in the effective potential as consequences of the local chiral transformation: one is the interaction term, $\bar{q}(\gamma_5\boldsymbol{\gamma} \cdot \nabla\theta/2)q$, due to the non-commutability of $\theta(\mathbf{r})$ with the kinetic (differential) operator in the Dirac operator. It may be easily seen that $\nabla\theta$ acts as an “external” axial-vector field for quarks and resolves the degeneracy of the single-particle energy spectra for different spin states. In this case, by a suitable rearrangement of the two Fermi seas we can always find an optimal configuration to lower the total energy. Another one is nontrivial and comes from the vacuum-polarization effect: the energy spectrum of quarks is modified in the presence of $\theta(\mathbf{r})$ and thereby the vacuum polarization should give an additional term, $\propto (\nabla\theta)^2$ in the lowest order. This can be regarded as an appearance of the kinetic energy term for DCDW through the vacuum polarization [13]. Thus, the Fermi sea works for DCDW, while the Dirac sea against it; when the interaction energy is superior to the kinetic energy, quark matter becomes unstable to form DCDW.

We will see that the mechanism is quite similar to that for the spin density wave suggested by Overhauser [16, 17], and entirely reflects many-body effects. In general, such a density wave is favored in 1-D (one spatial dimension) systems with the wave number $|\mathbf{q}| = 2p_F$ (p_F : Fermi momentum) according to the Peierls instability [14], e.g., charge density waves in quasi-1-D metals [15]. The essence of its mechanism is the nesting of Fermi surfaces and the level crossing of the single-particle energy spectra with a relative momentum \mathbf{q} . In the higher dimensional systems, however, the nesting is incomplete and the density wave should

be formed provided the interaction of a relevant (p - h) channel is strong enough. For the 3-D electron gas, it was shown by Overhauser [16, 17] that the paramagnetic state is unstable with respect to the formation of the static spin-density wave, in which the energy spectra of up- and down-spin states with a finite relative momentum exhibit a level crossing and it is resolved by the spin exchange interaction, while the wave number does not coincide with $2p_F$ completely because of the higher dimension.

We explicitly demonstrate that quark matter becomes unstable for a formation of DCDW at moderate densities, using the Nambu-Jona-Lasinio (NJL) model which is originally one of the schematic models to describe realization of chiral symmetry in the vacuum [1]. Recently the model has been also considered as an effective model of QCD embodying SSB of chiral symmetry in terms of quark degree of freedom [18].

We start with the NJL Lagrangian with $N_f = 2$ flavors and $N_c = 3$ colors,

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial - m_c)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2], \quad (2)$$

where m_c is the current mass, $m_c \simeq 5\text{MeV}$. Under the mean-field approximation, we keep the pseudo-scalar as well as the scalar mean-field. In the usual treatment to generate a dynamical quark mass and see the restoration of chiral symmetry within the NJL model, the pseudo-scalar mean-field is implicitly discarded. However, note that this is justified only for the vacuum since it is the definite eigenstate of parity, and there is no compelling reason at finite baryon density. We assume here the following form for the mean-fields,

$$\begin{aligned} \langle \bar{\psi}\psi \rangle &= \Delta \cos(\mathbf{q} \cdot \mathbf{r}) \\ \langle \bar{\psi}i\gamma_5\tau_3\psi \rangle &= \Delta \sin(\mathbf{q} \cdot \mathbf{r}), \end{aligned} \quad (3)$$

in the direct channel. Accordingly, we define a new quark field ψ_W by the Weinberg transformation,

$$\psi_W = \exp[i\gamma_5\tau_3\mathbf{q} \cdot \mathbf{r}/2]\psi, \quad (4)$$

to separate the degrees of freedom of the amplitude and the phase of DCDW in the Lagrangian. In terms of the new field the effective Lagrangian renders

$$\mathcal{L}_{MF} = \bar{\psi}_W[i\partial - M - 1/2\gamma_5\tau_3\mathbf{q} \cdot \mathbf{r}]\psi_W - G\Delta^2 + \mathcal{L}_{SB}, \quad (5)$$

where \mathcal{L}_{SB} is a small residual term due to the current mass m_c ,

$\mathcal{L}_{SB} = -m_c\bar{\psi}_W[\exp(i\gamma_5\tau_3\mathbf{q} \cdot \mathbf{r}) - 1]\psi_W$, and we put $M \equiv m_c - 2G\Delta$ and $q^\mu = (0, \mathbf{q})$. In the

following, we take the chiral limit ($m_c = 0$) discarding \mathcal{L}_{SB} . The form given in (5) appears to be the same as that in the superconductor model of the vacuum except an “external” axial-vector field, \mathbf{q} , generated by the wave vector of DCDW [18]; the *amplitude* of DCDW generates the dynamical quark mass in this case. Accordingly we can see that our theory becomes trivial in the chiral-symmetry restored phase.

The Dirac equation for ψ_W then gives a *spatially uniform* solution, $\psi_W = u_W(p) \exp(ip \cdot \mathbf{r})$, with the eigenvalues

$$E^\pm(\mathbf{p}) = \sqrt{E_p^2 + |\mathbf{q}|^2/4 \pm \sqrt{(\mathbf{p} \cdot \mathbf{q})^2 + M^2|\mathbf{q}|^2}}, \quad E_p = (M^2 + |\mathbf{p}|^2)^{1/2} \quad (6)$$

for positive energy (valence) quarks with different polarizations denoted by the sign \pm . For negative energy quarks in the Dirac sea, they have an energy spectrum symmetric with respect to null line because of charge conjugation symmetry in the Lagrangian (5).

The single-particle energy spectra (6) exhibit an analogue of the *exchange splitting* between energies with different spin states in the Stoner model [7]. Hereafter, we choose \mathbf{q}/\hat{z} , $\mathbf{q} = (0, 0, q)$ with $q > 0$, without loss of generality. Note that we need not distinguish two flavors in the energy spectra, while their eigenspinors are different to each other. The energy spectra (6) show a salient feature: they break rotation symmetry due to the coupling term of the momentum and the wave vector, and lead to the axially-symmetric deformed Fermi seas.

The effective potential then reads

$$\begin{aligned} \Omega_{\text{total}} &= \gamma \int \frac{d^3 p}{(2\pi)^3} \left[(E_p^- - \mu)\theta_- + (E_p^+ - \mu)\theta_+ \right] - \gamma \int \frac{d^3 p}{(2\pi)^3} \left[E_p^- + E_p^+ \right] + (M - m_c)^2/4G \\ &\equiv \Omega_{\text{val}} + \Omega_{\text{vac}} + M^2/4G \end{aligned} \quad (7)$$

after summing up all the energy levels, where $\theta_\pm = \theta(\mu - E_p^\pm)$, μ is the chemical potential and γ the degeneracy factor $\gamma = N_f N_c (= 6)$. The first term Ω_{val} is the contribution by the valence quarks filled up to the chemical potential μ in each Fermi sea, while the second term Ω_{vac} is the vacuum contribution that is formally divergent. We shall see both contributions are *indispensable* in our context. Since the NJL model is nonrenormalizable, we need some regularization procedure to extract a meaningful finite value for the vacuum contribution. Note that we cannot apply the usual momentum cut-off regularization (MCOR) scheme to Ω_{vac} , since the energy spectrum has no more rotation symmetry. Instead, we adopt the proper-time regularization (PTR) scheme [19], which may be a most suitable one for our

purpose, since Ω_{vac} counts the vacuum-polarization contributions under the “external” axial-vector field \mathbf{q} . Incidentally the vacuum polarization effect under the external electromagnetic field has been treated in a gauge invariant way by using the PTR scheme, where the energy spectrum is also deformed depending on the field strength [19].

The vacuum contribution Ω_{vac} can be represented as

$$\Omega_{\text{vac}} = i\gamma \int \frac{d^4 p}{(2\pi)^4} \text{trln}(S_W(p)/S_0(p)), \quad (8)$$

where $S_W(p)$ is the quark propagator, $S_W(p) = [\not{p} - M - 1/2\gamma_5\tau_3\not{q}]^{-1}$. Here we subtracted an irrelevant constant $\Omega_{\text{ref}} = i\gamma \int d^4 p/(2\pi)^4 \text{trln}(S_0(p))$ with $S_0(p) = [\not{p} - m_{\text{ref}}]^{-1}$ to make the following procedure mathematically well-defined, which is the quark propagator with an arbitrary mass m_{ref} . Introducing the proper-time variable τ , we eventually find

$$\Omega_{\text{vac}} = \frac{\gamma}{8\pi^{3/2}} \int_0^\infty \frac{d\tau}{\tau^{5/2}} \int_{-\infty}^\infty \frac{dp_z}{2\pi} \left[e^{-(\sqrt{p_z^2 + M^2} + q/2)^2\tau} + e^{-(\sqrt{p_z^2 + M^2} - q/2)^2\tau} \right] - \Omega_{\text{ref}}, \quad (9)$$

which is obviously reduced to the standard formula [18] in the limit $q \rightarrow 0$ (no DCDW). The integral with respect to τ is not well defined yet as it is, since it is still divergent due to the $\tau \simeq 0$ contribution. Regularization further proceeds by replacing the lower bound of the integration range by $1/\Lambda^2$, which corresponds to the momentum cut-off in the MCOR scheme.

Now we examine a possible instability of quark matter with respect to formation of DCDW. For the vacuum contribution, we can easily see $\Omega_{\text{vac}} \geq \Omega_{\text{vac}}|_{q \rightarrow 0}$, which means the vacuum (the Dirac sea) is stable against formation of DCDW, as it should be. For small q ,

$$\Omega_{\text{vac}} = \Omega_{\text{vac}}|_{q \rightarrow 0} + \frac{\gamma\Lambda^2}{16\pi^2} J(M^2/\Lambda^2)q^2 + O(q^4) \quad (10)$$

where $J(x)$ is a function, $J(x) = -x\text{Ei}(-x)$, with the exponential integral $\text{Ei}(-x)$. The coefficient of q^2 term in Ω_{vac} , which is called the spin stiffness parameter β_{vac} , has a definite physical meaning: since the pion decay constant f_π is given in terms of $J(x)$ within the NJL model [18],

$$f_\pi^2 = \frac{\gamma\Lambda^2}{8\pi^2} J(M^2/\Lambda^2), \quad (11)$$

β_{vac} can be written as $\beta_{\text{vac}} = \frac{1}{2}f_\pi^2$. There are two remarks in order. One is that there appears no divergence as $\Lambda \rightarrow \infty$ in the higher order terms in q^2 . Hence, strictly speaking, higher order terms should be discarded [13], since the divergent term should dominate over other finite terms. However, we keep them here to preserve self-consistency after introduction

of the regularization in the nonrenormalizable theory. The other is that Eq. (10) suggests that the vacuum-polarization effect precisely provides the expected kinetic-energy term for DCDW, except a “running” pion decay constant (11).

For given μ, M and q we can evaluate the valence contribution Ω_{val} using Eq. (6) and write down the general formula analytically. For small q we have a following expansion,

$$\Omega_{\text{val}} = \Omega_{\text{val}}|_{q \rightarrow 0} - \frac{\gamma}{8\pi^2} M^2 q^2 H(\mu/M) + O(q^4) \quad (12)$$

where $\Omega_{\text{val}}|_{q \rightarrow 0} = \gamma/(24\pi^2) \left[\mu\sqrt{\mu^2 - M^2} (5M^2 - 2\mu^2) - 3M^4 \ln(\mu + \sqrt{\mu^2 - M^2}/m) \right]$ for normal quark matter and $H(x) = \ln(x + \sqrt{x^2 - 1})$. The spin stiffness parameter then reads

$$\beta_{\text{val}} = -\frac{\gamma}{8\pi^2} M^2 H(\mu/M) \quad (13)$$

for valence quarks. Since the function $H(x)$ is always positive and a monotonously increasing function, we can see $\beta_{\text{val}} \leq 0$.

The total spin-stiffness parameter, $\beta_{\text{tot}} \equiv \beta_{\text{val}} + \beta_{\text{vac}}$, depends on the dynamical mass and the chemical potential for a given Λ . In the vacuum it is positive, and the necessary condition to form DCDW then reads $\beta_{\text{tot}} < 0$ due to the finite-density effect. When this condition is fulfilled, the optimal value of q is determined by the minimum of the effective potential (7). We can see that the optimal value of q is $O(2p_F)$ with the quark Fermi momentum p_F without recourse to explicit calculations. First, consider the energy spectra for massless quarks (see Fig.1). As is already discussed, our theory becomes trivial in this case and we find two energy spectra

$$E^\pm(\mathbf{p}) = \sqrt{p_\perp^2 + (|p_z| \pm q/2)^2}, \quad \mathbf{p}_\perp = (p_x, p_y, 0), \quad (14)$$

essentially equivalent to the usual ones $E^\pm(\mathbf{p}) = |\mathbf{p}| (\equiv E(\mathbf{p}))$; taking a linear combination of the eigenspinors ψ_W^\pm , we can reconstruct the eigenstates of definite chirality γ_5 to get the energy spectra, $\tilde{E}^\pm(\mathbf{p}) = \sqrt{p_\perp^2 + (p_z \pm q/2)^2}$. Then we can see a level crossing on the $p_z = 0$ plane. Once the mass term is taken into account this degeneracy is resolved and the energy gap appears there. The resultant spectra are given by (6). Hence we have always an energy gain by filling only the lower-energy spectrum $E^-(\mathbf{p})$ up to the Fermi energy, if the relation $,q = O(2p_F)$, holds. Thus, we find that this mechanism is very similar to that of spin density wave by Overhauser [16, 17] [23].

Taking the extremum of the effective potential (7) with respect to the order-parameters M and q , we can determine their values for given baryon-number densities. Fig. 2 demonstrates

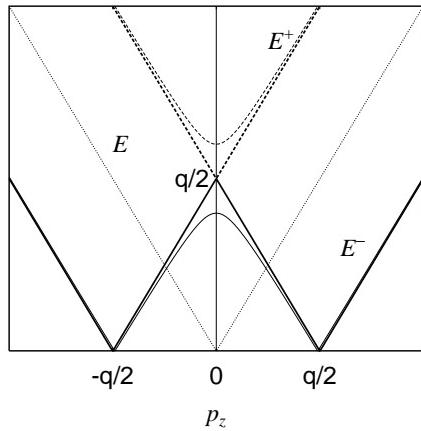


FIG. 1: Energy spectra for $\mathbf{p}_\perp = 0$. E^\pm with $M = 0$ (thick solid and dashed lines). \tilde{E}^\pm with the definite chirality is also shown for comparison (dotted line). We can see there is a degeneracy of E^\pm at $p_z = 0$ for $M = 0$, while it is resolved by the mass (thin solid and dashed lines).

the behaviors of the optimum values as functions of μ at zero temperature for a parameter set, $G\Lambda^2 = 6$, $\Lambda = 860$ MeV. It is found that the magnitude of q becomes finite just before

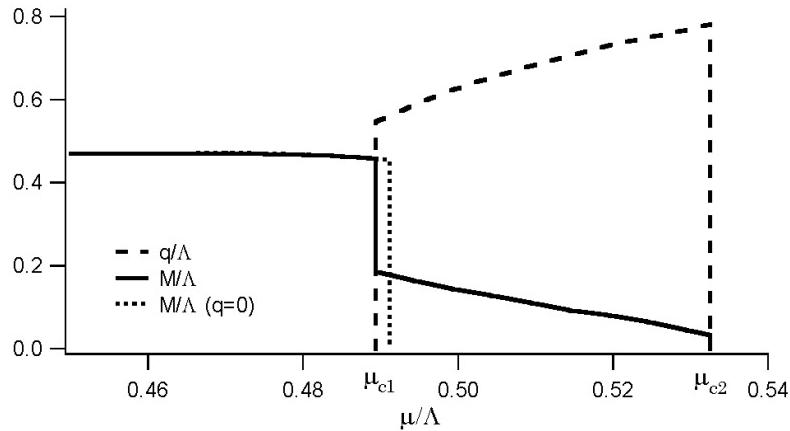


FIG. 2: The wave number q and the dynamical mass M are plotted as functions of the chemical potential at zero temperature. Solid (dotted) line for M with (without) DCDW, and dashed line for q .

the ordinary chiral-symmetry restoration, and DCDW survives in the finite range of μ , $\mu_{c1} \leq \mu \leq \mu_{c2}$, which corresponds to the baryon-number densities $\rho_b/\rho_0 = 3.62 - 5.30$. The dynamical mass remains finite in this density region, which means chiral restoration is

delayed in the presence of DCDW. The wave number q increases with μ , while its value is smaller than $2p_F$ ($\simeq 2\mu$ for massless quarks) due to the higher dimensional effects. Actually, the ratio of the wave number and the Fermi momentum (at normal phase $q = M = 0$) lies in the range, $q/p_F = 1.17 - 1.47$. We thus conclude that DCDW is induced by finite-density contributions, and has the effect to extend the SSB phase ($M \neq 0$) to high density region, which suggests another path for chiral-symmetry restoration by way of the DCDW state at finite density.

Here we would like to point out an interesting aspect of the DCDW state. The quark-DCDW coupling is spin dependent and we can see that it gives rise to a spatial oscillation of the magnetic moment in quark matter. The magnetic moment is given as

$$M_z \propto \langle \bar{\psi} \sigma_{12} \psi \rangle = M_z^W \cos(\mathbf{q} \cdot \mathbf{r}), \quad (15)$$

where $M_z^W (= \langle \bar{\psi}_W \sigma_{12} \psi_W \rangle)$ is proportional to the dynamical mass. Thus DCDW can be regarded as a kind of the *spin density wave* [16, 17] and should have phenomenological implications on the magnetic properties of compact stars.

In this Letter we have only considered the direct channel (Hartree term) of the four Fermi interaction. When we consider the exchange contributions (Fock term), there appear additional quark-quark interactions in the vector and axial-vector channels by way of the Fierz transformation [8, 20]. It is well known that the vector contribution is rather trivial and summed up as a renormalization of the chemical potential. We are then interested in the axial-vector contribution in our context. We may see that it renormalizes the wave vector of DCDW; this can be regarded as a kind of vertex renormalization of the quark-DCDW coupling. This subject will be discussed elsewhere [21].

It would be interesting to recall that DCDW is similar to pion condensation within the σ model, considered by Dautry and Nyman [22], where σ and π^0 meson condensates take the same form as Eq. (1). So it might be possible to connect pion condensation before deconfinement with DCDW after it by a symmetry consideration. However, the restoration of chiral symmetry, especially the vacuum contribution, has been poorly considered in the context of pion condensation.

Finally, it might be interesting to generalize the DCDW configuration (3) by incorporating the radial degree of freedom; one may generalize the amplitude to be spatially dependent as well à la refs.[9, 10], but one must consider a genuine *non-uniform* matter in this case.

Anyhow, if two degrees of freedom work coherently, we may have more favorable configurations than considered here.

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- [1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345.
- [2] D. Bailin and A. Love, Phys. Reports 107(1984) 325.
For a review, e.g., M. Alford, Ann. Rev. Nucl. Part. Sci. 51 (2001) 131.
- [3] J. Berges and K. Rajagopal, Nucl. Phys. **B538** (1999) 215.
R. Rapp, T. Schaefer, E.V. Shuryak, M. Velkovsky, Phys. Rev. Lett. 81 (1998) 53.
- [4] T. Tatsumi, Phys. Lett. **B489** (2000) 280.
- [5] A. Niégawa, hep-ph/0404252.
- [6] For a recent review, P.M. Woods and C. Thompson, astro-ph/0406133.
- [7] F. Bloch, Z. Phys. **57** (1929) 545.
K. Yosida, *Theory of magnetism* (Springer, 1998).
- [8] E. Nakano, T. Maruyama and T. Tatsumi, Phys. Rev. **D68** (2003) 105001.
- [9] E. Shuster and D. T. Son, Nucl. Phys. **B573** (2000) 434.
- [10] B.-Y. Park, M.Rho, A.Wirzba and I.Zahed, Phys. Rev. **D62** (2000) 034015.
R. Rapp, E.Shuryak and I. Zahed, Phys. Rev. **D63** (2001) 034008.
- [11] D.V. Deryagin, D. Yu. Grigoriev and V.A. Rubakov, Int. J. Mod. Phys. **A7** (1992) 659.
- [12] S. Weinberg, *The quantum theory of field II* (Cambridge, 1996).
- [13] T. Eguchi and H. Sugawara, Phys. Rev. **D10** (1974) 4257.
K. Kikkawa, Prog. Theor. Phys. **56** (1974) 947.
- [14] R. E. Peierls, *Quantum Theory of Solids* (Oxford University Press, London, 1955).
- [15] G. Grüner, Rev. Mod. Phys. **60** (1988) 4.
- [16] A.W. Overhauser, Phys. Rev. Lett. **4** (1960) 462; Phys. Rev. **128** (1962) 1437.
- [17] G. Grüner, Rev. Mod. Phys. **66** (1994) 1.

- [18] S.P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.
T. Hatsuda and T. Kunihiro, Phys. Rep. **247** (1994) 221.
- [19] J. Schwinger, Phys. Rev. **92** (1951) 664.
- [20] T. Maruyama and T. Tatsumi, Nucl. Phys. **A693** (2001) 710.
- [21] E. Nakano and T. Tatsumi, in preparation.
- [22] F. Dautry and E.M. Nyman, Nucl. Phys. **319** (1979) 323.
K. Takahashi and T. Tatsumi, Phys. Rev. **C63** (2000) 015205; Prog. Theor. Phys. **105** (2001) 437.
M. Kutschera, W. Broniowski and A. Kotlorz, Nucl. Phys. **A516** (1990) 566.
- [23] It has been shown that similar mechanism by valence quarks also works in another context of the chiral density wave [9, 10].